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Strategy Discovery as a Competitive Negotiation between Metacognitive and Associative Mechanisms

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Both metacognitive and associative models have been proposed to account for children's strategy discovery and use. Models based on only metacognitive or only associative mechanisms cannot entirely account for the observed mix of variability and constraint revealed by recent microgenetic studies of children's strategy change. We propose a new approach where metacognitive and associative mechanisms interact in a competitive negotiation. This approach provides the flexibility to model the observed variability and constraint.

A variety of mechanisms have been proposed to account for strategy discovery in both adults and children. Most can be classified into one of two groups: Metacognitive and associative mechanisms. Metacognitive mechanisms provide the representations and processes necessary to explain the part of human cognition that is explicit, flexible, and responsive to problem-

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solving goals. Associative approaches provide representations and processes suited to explaining the part of human cognition that is implicit, fast, and responsive to nuances of the environment. Although precise definitions have varied, those interested in strategy use have long been concerned with describing the ways that these two classes of mechanisms interact in the discovery and generalization of problem solving strategies (e.g., Flavell, Miller, & Miller, 1993; Greeno, Riley, Gelman, 1984; Hiebert & LeFevre, 1993; Karmiloff-Smith, 1992; Piaget, 1976, 1978; Schneider & Pressley, 1989).

In this article we revisit the question of how metacognitive and associative mechanisms interact while children learn new problem solving strategies. We begin by describing the properties of the two groups of mechanisms and the behavior that would be expected to be observed in a child using metacognitive or associative mechanisms to discover a new strategy. We then review empirical and computational evidence suggesting that neither group of mechanisms alone can account for a particular strategy discovery on which a substantial database exists: young children's invention of the min strategy for solving simple arithmetic problems. Based on consideration of this database, we develop a new account of strategy discovery, one in which metacognitive and associative mechanisms are coordinated through a process of competitive negotiation. We believe that this approach can provide both the sensitivity to environmental influences and the flexibility in adopting problem solving goals that are needed to model strategy discovery processes in many domains.

STRATEGY DISCOVERY THROUGH METACOGNITIVE AND ASSOCIATIVE MECHANISMS

Metacognitive and associative mechanisms have distinct properties and thus make distinct predictions about how strategies are discovered and generalized. Metacognitive mechanisms share the assumption that strategies are invented and selected by children explicitly reflecting upon their understanding of task demands, available cognitive resources, and their own experience solving similar problems. As they gain experience in a domain, children accumulate an increasingly deep, well-organized store of explicit knowledge about their competencies and about the particulars of the task. Effective strategy use depends on the problem solver's ability to use this explicit knowledge to select the most adaptive strategies and, if no existing strategy is deemed appropriate, to plan a new problem solving approach.

Associative mechanisms are defined by the assumption that strategy selection is determined by a set of learned correlations among tasks, actions, and outcomes. Usually, the correlational knowledge is implicit rather than explicit. During problem solving, actions compete with one another on the basis of how strongly they are associated with features of the particular task and with past outcomes. If the chosen actions lead to success, the links between them and the task will be strengthened, making it more likely that they will be chosen in the future. Within this approach, expert strategy users are those who have a well-tuned set of associations that automatically activate the best strategies for a wide range of tasks. Because statistical associations can only be tuned by experience, what distinguishes experts from novices is primarily the breadth and depth of the expert's problem-solving experience in a particular domain.

The different representations and processes associated with metacognitive and associative mechanisms produce concomitant differences in accessibility, flexibility, and speed. Because metacognitive knowledge is potentially verbalizable, it can be accessed and modified through reflective mechanisms. It can be used to construct detailed plans for future action or to debug past failures. It also can be argued about with collaborators or learned from reading a book. Because direct problem-solving experience is not necessary to access and modify metacognitive knowledge, it enables children to reason their way to solutions in novel contexts, without the need to engage in trial-and-error search. This flexibility comes at a price, however. Metacognitive mechanisms, such as planning or means-ends analysis, are inherently serial and slow. They also are resource-intensive; even on simple tasks, the cognitive demands of such reasoning can push working memory to its limits, or beyond (Carpenter, Just, & Shell, 1990).

Associative mechanisms have complementary strengths and weaknesses. Because problem-solving experience is summarized in an association matrix or a network of connection strengths, associative knowledge provides rapid and adaptive fits between strategies and problem-solving contexts. Because they operate without the need for reflective awareness, associative mechanisms allow for the automatization of skills and the consequent freeing up of processing resources. However, because the associative knowledge base is largely implicit, it cannot be directly examined or modified through metacognitive mechanisms. Therefore, associative learning requires extensive problem solving experience. Also, because associative learning tends to be quite specific, it does not provide a useful base for coping flexibly with novel contexts. To discover new strategies in such contexts, a problem solver relying only on associative mechanisms would have little recourse but to fall back on trial-and-error search (Holland, Holyoak, Nisbett, & Thagard, 1986).

Few would argue that children's strategy use could be accounted for by the operation of only metacognitive or associative mechanisms. Each mechanism captures something essential about children's thinking, and each almost certainly plays a role in the discovery and generalization of problem solving strategies. The crucial unresolved issues concern what role each plays and how these roles are coordinated.

One influential attempt to deal with these issues is the representational translation account pioneered by Piaget (1976; 1978) and later championed by Karmiloff-Smith (1992). Common to both theorists is the argument that problem solving knowledge exists initially in an implicit procedural representation and is later translated into an explicit conceptual representation. Within these accounts, when children are novices in a domain, they engage in trial-and-error experimentation to see what works. If they stumble across an action that leads to a successful solution, they more frequently use that action on future problems. Through repeated trial-and-error experience, children build a repertoire of successful strategies governed by associative mechanisms that require no explicit, metacognitive input. Once children are reliably successful, they begin a process of ‘‘representational redescription’’ in which they translate their previously implicit strategies into an explicit representation that can be accessed through reflective metacognitive processes. By gaining reflective access to the strategy, children become able to adapt and generalize it in new contexts.

A second approach to coordinating metacognitive and associative knowledge can be seen in models constructed to explain the automatization of problem solving skills in both adults and children (e.g., Anderson, 1990). Translation in these approaches flows in the opposite direction of that proposed by Karmiloff-Smith. Initial strategies are seen as explicit plans of action whose successful execution requires high levels of intentional metacognitive control. Once children have sufficient practice in the domain, the strategy is compiled into an efficient procedural representation that can be activated and executed without the need for metacognitive micromanagement. Often, this translation process is seen as overwriting the original metacognitive version of the strategy. Supporting this view are the many tasks on which we are sufficiently expert that we can no longer describe the strategies verbally. For example, almost all adults can tie their shoes, but to teach a child how to do it, they have to reinvent a verbal description of the skill.

Both Karmiloff-Smith’s and Anderson’s approaches recognize that using problem solving strategies is sometimes explicit and effortful and sometimes implicit and resource-lean. Although the two approaches differ in whether translation occurs from metacognitive to associative knowledge, or in the opposite direction, both share an important assumption: Metacognitive and associative knowledge do not compete with one another for control of problem solving. One group of mechanisms is responsible for discovering a strategy and the other group of mechanisms is responsible for generalizing it. Translation passes control from one group of mechanisms to the other.

The two versions of the translation story make opposite predictions about the behavior that should be expected to accompany children’s discovery of new strategies. Because children discover new strategies through trial-and-error in the associative-to-metacognitive version, one would predict that:

1. Discoveries should occur when children are at impasses and do not have appropriate strategies for solving the problems (which is why they resort to trial and error);

2. Discoveries should not involve metacognitive insight (which only comes later);

3. Children will try both successful strategies and unsuccessful strategies (because they are relying on trial and error).

In the metacognitive-to-associative version of strategy discovery, the implications of the discovery through reflective, explicit reasoning are that:

1. Discoveries can occur at impasses, but they also could occur through noticing something interesting about prior solutions or through direct instruction by another problem solver;
2. Discoveries should be accompanied by explicit, metacognitive knowledge (because the strategy is originally represented in explicit, declarative form);
3. Discoveries could be restricted to only successful strategies (because explicit reasoning allows problem solvers to make predictions about the success of a new strategy before they commit to trying it out).

IS STRATEGY DISCOVERY METACOGNITIVE OR ASSOCIATIVE?

How well do these contrasting accounts of strategy discovery fit the empirical evidence? One discovery on which a sufficient database exists to answer this question is children's invention of the min strategy for adding numbers. By the time they enter kindergarten, most children use multiple strategies to solve addition problems (Siegler & Shrager, 1984). Their single most common approach is the *sum* strategy, where they first count out each addend on their fingers and then count all of the raised fingers to get the sum. For example, to solve 2 + 3, a child would count, ''1,2 . . . 1,2,3 . . . 1,2,3,4,5.'' By the middle of first grade, most children discover the *min* strategy, where they represent the larger addend by simply saying it and then counting up from it the number of times indicated by the smaller addend. For 2 + 3, a child would count ''3,4,5'' or ''4,5.''

What does children's behavior look like when they make this discovery? Siegler and Jenkins (1989) identified 4- and 5-year-olds who knew how to add via the sum strategy but who did not yet know the min strategy. Over an 11-week period, these children were videotaped as they participated in roughly 30 sessions each of addition problem solving. After solving each problem, children were asked to explain how they came to their solution. By analyzing these immediate retrospective verbal reports in concert with the videotaped records of children's ongoing behavior, it was possible to obtain accurate assessments of the strategy that children used to solve each addition problem (Siegler, 1987).

These trial-by-trial strategy assessments made it possible to identify the exact trial on which children first used the min strategy. By analyzing children's behavior before, during, and after their discovery of the min strategy, Siegler and Jenkins generated an empirical profile for min discovery that did not correspond completely with the predictions of either the metacognitive or the associative accounts of strategy discovery. As we will describe, in some aspects of discovery where existing approaches would predict regularity, Siegler and Jenkins found variability; in other aspects where existing approaches would predict variability, Siegler and Jenkins found regularity. The profile of strategy discovery we present below is not limited to Siegler and Jenkins' study, but is consistent with a more recent study of min discovery from an unrelated laboratory (Ward, Hawk, & Grupe, 1995) as well as with findings from studies of discovery in domains other than simple arithmetic (Siegler, 1996; Siegler & Crowley, 1991).

Discoveries Occurred at Varied Times and on Varied Problems

Children began the study with similar knowledge of addition, they encountered the same problems during the study, and yet they discovered the min strategy in widely varying ways. Through pretesting, Siegler and Jenkins ensured that children who participated in the study did so from approximately the same starting point. None of the children used the min strategy, but they did use other strategies well enough to generate the correct answer on at least 50% of addition problems with addends 1–5. Further, all children already possessed high levels of competence in counting and magnitude comparison, two prerequisite skills for construction of the min strategy.

Despite the similarity of their initial knowledge, there was little uniformity in when children discovered the min strategy. The discoveries occurred from the 2nd to the 22nd session; in real time, the range was 2–73 days. The discoveries occurred on disparate problems. The seven children who discovered the min strategy did so on 6 different problems, ranging from easy problems ($4 + 1$) to difficult ones ($3 + 9$).

Even in examining the sequence of learning of individual children, there was nothing atypical about the problems or the context in which the min strategy first emerged. Discoveries did not typically occur on problems where the child was at an impasse; the particular problems on which discoveries occurred were not unusually difficult, and children's success on prior problems within the same session was comparable to their success in the study as a whole. Moreover, children often first used the min strategy on a problem that they previously solved correctly using a different addition strategy.

Discoveries Occurred with Varying Degree of Reflective Insight

After children solved each addition problem, the experimenter asked them to explain how they had computed the answer. Examination of these immediate retrospective protocols revealed that on the trial where children first used the min strategy, there was considerable variability in how much insight children had into the strategy they had just discovered. In self-reports given immediately after the first trial where they used the min strategy, roughly half of the children showed metacognitive awareness of the new strategy. Typifying this insight was Brittany's protocol:

E: OK Brittany, how much is 2 + 5?

B: 2 + 5—[whispers] 6, 7—it's 7.

E: How did you know that?

B: [excitedly] Never counted.

E: You didn't count?

B: Just said it—I just said after 6 something—7—6, 7.

E: You did? Why did you say 6, 7?

B: Cause I wanted to see what it really was.

E: OK, well—so, did you—what—you didn't have to start at one, you didn't count 1, 2, 3, you just said 6, 7?

B: Yeah—smart answer.

However, the other half of the children showed little or no explicit understanding of their newly discovered strategy. Some of them gave confused, fragmented explanations that referred only indirectly to the strategy they had used. Others showed even clearer evidence that they did not explicitly understand what they had just done. Whitney's protocol was typical of this subgroup. On the videotape, she can be clearly seen using the min strategy while counting out on her fingers. However, when asked to explain how she computed the answer, she denied that she had counted out an answer at all:

E: How much is 4 + 3?

W: 5, 6, 7, I think it's 7.

E: 7, OK, how did you know that?

W: Because I'm smart and I just knew it.

E: Can you tell me, I heard you counting. I hear you. Tell me how you counted.

W: I just—I didn't count anything—[long pause] I just added numbers onto it.

E: Can you tell me how you added numbers?

W: No.

Greater Reflective Insight Was Associated with Faster Generalization of the New Strategy

After children discovered the min strategy, they generalized it to other problems surprisingly slowly. In the five sessions that followed their discovery of the strategy, children used the new approach on only 12% of the trials on which they counted out answers. On the remaining 88% of trials on which they counted, they continued to use less efficient strategies such as counting from one.

The failure of children to immediately generalize the min strategy suggests that generalization of the new strategy was determined by associative mechanisms. Even children such as Brittany, who explicitly noted that the new strategy was smart, did not use it very often. The limited generalization presumably occurred because the associations between the new strategy and arithmetic problems were not strong enough to allow the strategy to reliably win competitions with well-established approaches such as the sum strategy. After repeated successful use, the min strategy's associations with the task presumably become stronger and come to dominate the sum strategy. The min strategy did, in fact, emerge as the dominant counting strategy by the end of the Siegler and Jenkins study.

However, the evidence suggested that generalization was not exclusively influenced by associative mechanisms. Recall that at the moment when they discovered the min strategy, children exhibited variability in how much insight they had into the workings of their new creation. Those who had shown the greatest level of explicit insight at the moment of discovery generalized the strategy faster and more completely (min constituted more than 40% of all subsequent uses of counting strategies) than children who had shown the least insight (min constituted less than 10% of subsequent uses of a counting strategy). All children still exhibited uneven generalization, but metacognitive awareness appeared to accelerate the generalization process.

Discoveries Were Constrained in Ways That Avoided Illegitimate Strategies

In the midst of the diversity in when and how the min strategy was discovered, one striking regularity emerged: Children never discovered illegitimate addition strategies. By illegitimate strategies, we mean strategies that violate the necessary goal structure of the domain. In the case of addition, all legitimate strategies are strategies that accomplish three subgoals: (1) quantitatively represent the first addend, (2) quantitatively represent the second addend, and (3) quantitatively represent the combined set of both addends. Legitimate addition strategies vary in how they satisfy these three subgoals. For example, in the sum strategy, all of the subgoals are met by counting; in the min strategy, only some of the subgoals are met by counting; in retrieval, no counting is required to meet the subgoals. However, without appropriate procedures to satisfy each subgoal, no addition strategy could be successful.

When considered along with children's self-reports regarding their strategy use, this finding suggests that children's discovery of the min strategy involved at least two distinct sets of mechanisms. One way to generate only legitimate strategies is through metacognitive analysis. If children had insight into the goal structure of a legitimate addition strategy, they could have constructed new strategies by reflecting upon the potential for different procedures to meet each of the three essential addition subgoals. With sufficient metacognitive knowledge and the ability to make accurate projections of the potential of a new strategy, children would be expected to only try legitimate strategies. However, when asked to explain their newly discovered min strategy, only about half of the children gave evidence that they had access to the type of explicit understanding that such metacognitive analysis would require.[1]

The opaque explanations given by the remaining children suggested that their discovery of the min strategy had not been the result of high-level metacognitive analysis. However, the evidence was also not consistent with discovery through associative mechanisms alone. If children possessed only associative knowledge of how well existing strategies worked on problems, they would need to discover new strategies via trial-and-error. Such an approach would almost certainly lead at least some children sometimes to combine familiar addition procedures into illegitimate strategies. Yet surprisingly, despite their lack of awareness, these children managed to discover the min strategy without ever trying flawed strategies.

Siegler and Jenkins (1989) suggested that this lack of trial and error was possible because the preschoolers possessed a *goal sketch* for simple arithmetic. The goal sketch was hypothesized to specify the subgoals that all legitimate arithmetic strategies must satisfy, thus directing discovery processes away from procedures that would fail to satisfy these essential subgoals and toward procedures that would succeed. Consistent with this hypothesis, a later experiment demonstrated that kindergartners who did not yet use the min strategy nonetheless recognized it as legitimate, even though they could not verbalize a rationale for their judgment (Siegler & Crowley, 1994). The same children recognized that a strategy that did not meet all of the subgoals was inferior. However, none of the children could state why the one unfamiliar strategy was smart and the other not smart. Thus, children can evaluate alternative novel addition strategies on the basis of implicit knowledge; such evaluations seem likely to contribute to their not needing to try flawed strategies before discovering the min approach. Accounting for this surprising ability was a major motivation for developing the account of strategy discovery presented in this article.

COMPUTATIONAL ACCOUNTS OF STRATEGY DISCOVERY

The Siegler and Jenkins (1989) findings suggest that discovering the min strategy may involve more than metacognitive or associative mechanisms working alone. Two computer simulation models provide support for this conclusion. One model begins knowing the sum strategy and discovers the min strategy through the use of top-down metacognitive reasoning (Neches,

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[1] Evidence of explicit understanding of the min strategy should not be interpreted as conclusive evidence that children used metacognitive mechanisms to discover the strategy. Discoveries could have been generated by other means and then interpreted moments later at the metacognitive level. However, existence of explicit understanding does suggest that metacognitive mechanisms could have participated in the discovery, since children could express sufficient metacognitive understanding so close to the moment of discovery.

1987). The other simulation makes the sum-to-min transition primarily through associative mechanisms (Jones & VanLehn, 1991). The performance produced by these models suggests that although metacognitive and associative mechanisms are each sufficient to invent the min strategy, the operation of these mechanisms necessarily produces behavior unlike that in children's discoveries.

A Task Analysis of the Sum-to-Min Transition

The first step in modeling the sum-to-min transition is to represent the two strategies in terms of their component procedures and to determine the changes in these procedures that would be necessary for children who know the sum strategy to discover the min strategy.

In simulation models built to explain how children adaptively choose among existing strategies, the constituent procedures of a single strategy are typically represented as encapsulated chains of action that are activated, executed, and updated as monolithic entities (Siegler & Shrager, 1984; Siegler & Shipley, 1995). Once a strategy is chosen for execution, it runs from beginning to end without pause. After success or failure, credit or blame is assigned to the entire strategy. Individual procedures within each strategy are never directly accessed or modified. When the goal of a model is to explain how choices change between established competing strategies, it is appropriate to make strategies the unit of analysis.

However, if we are to explain not just how choices among strategies change, but also how strategies are created, the most appropriate unit of analysis is component procedures from which new strategies can be created. Different strategies for a domain share many of the same procedural components. For example, the sum and min strategies both depend on counting procedures that are established problem solving skills in their own right, skills that exist before children begin to solve arithmetic problems (Gelman & Gallistel, 1978). It seems unlikely that each arithmetic strategy maintains independent counting procedures. Instead, strategies that depend on similar procedures probably access these procedures during their execution. This shifts the definition of strategies from a single, static chain of procedures that can be activate with a single choice, to dynamic assemblies of procedures whose successful execution requires a series of choices about which procedure to use next.

Figure 1 outlines how changes among component procedures could enable a child who knows the sum strategy to discover the min strategy. As shown in the figure, six procedural steps constitute the sum strategy: (1) assigning an addend to be represented first (A1); (2) assigning an addend to be represented second (A2); (3) counting out the value of A1; (4) counting out the value of A2; (5) counting out the part of the sum represented by A1; and (6) counting out the part of the sum represented by A2.

From this base of knowledge of the sum strategy, discovery of the min

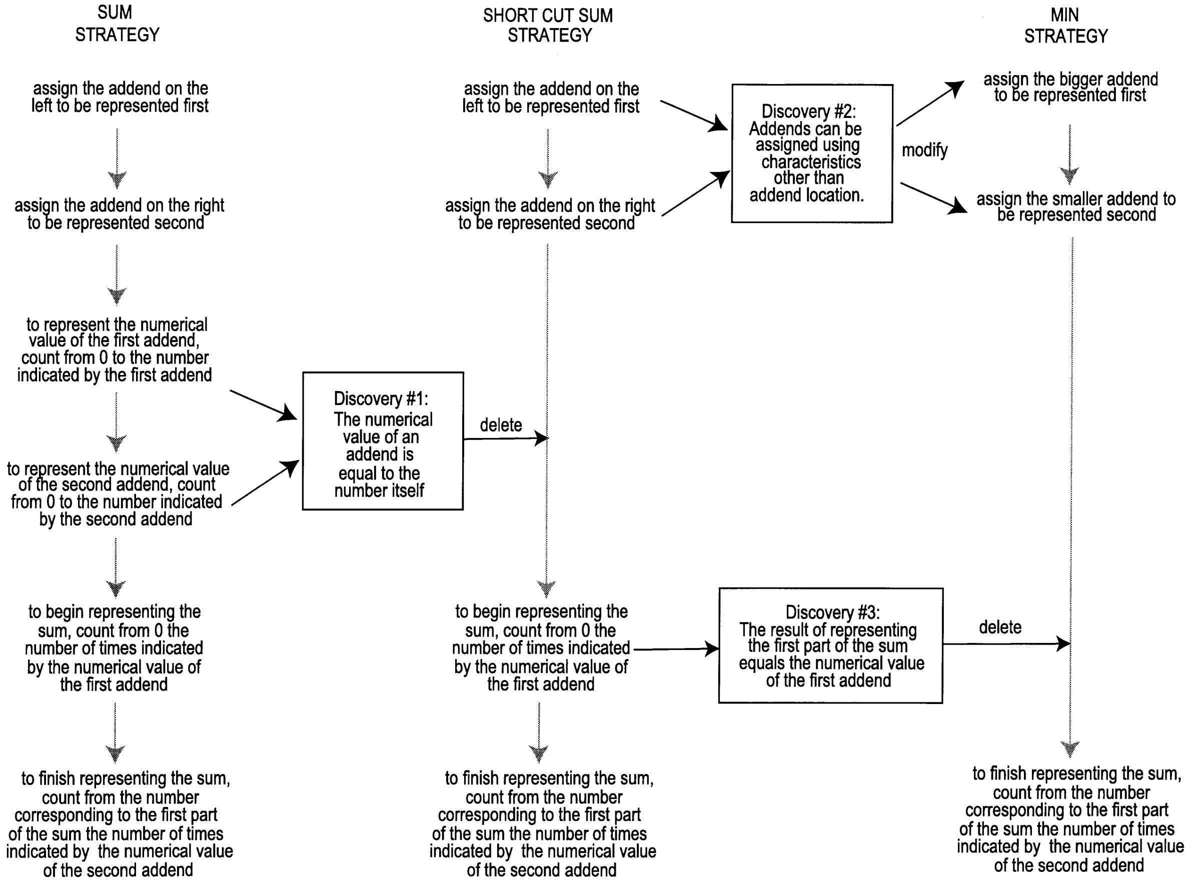

FIG. 1. The sum to min transition can be represented as three discoveries that delete and modify the steps that constitute each addition strategy.

strategy occurs in three steps. The first step is recognizing that Steps 3 and 4 are not necessary, because the counting always produces the original value of the addends. This discovery enables the problem solver to choose to bypass these steps. The resulting strategy—the shortcut sum approach—generates correct answers through Steps A1, A2, A5, and A6. Consistent with the view that this step occurs before discovery of the min strategy and is transitional to it, 5 of the 7 subjects who discovered the min strategy in Siegler and Jenkins (1989) discovered the shortcut sum approach either two sessions before they discovered the min strategy, one session before the discovery, or in the same session.

Two further discoveries enable the problem solver who knows the shortcut sum strategy to discover the min strategy. First is the discovery that the order in which addends are chosen to be represented does not need to conform to the order in which they are stated. This realization enables the problem solver to represent the larger addend first, regardless of whether it is first or second in the original problem. This enhances the efficiency of both the shortcut sum and, later, the min strategy. Second, the problem solver needs to discover that counting out the value of A1 (the original Step 5) always results in a subtotal equal to the value of A1. This enables the problem solver to delete the step that counts out the part of the sum represented by A1. Having made these three discoveries, the problem solver can use the min strategy.

This path from sum to min was the one followed by most children in Siegler and Jenkins (1989). Although all three discoveries seem necessary, they do not need to be made in the order depicted in Fig. 1. For example, some children made the discovery that addend order can be reversed in the context of the sum strategy, and reversed addend order before using either the shortcut sum or min strategies.

Although the Siegler and Jenkins data indicate the typical path through which children discover the min strategy, they do not specify the mechanisms that generated that path. It is because empirical methods can only reveal so much that psychologists create computer simulation models. These models test hypothesized mechanisms to see whether they are sufficient to produce the observed behavior. We next examine the performance of two simulations of discovery of the min strategy to see how well they meet this test.

HPM: A Metacognitive Account of Strategy Discovery

In Neches' *H*euristic *P*rocedure *M*odification (HPM) system, the min strategy emerges as a result of the operation of reasoning heuristics that ''represent metaknowledge about interesting properties of procedures'' (Neches, 1987, p. 214). Among metacognitive accounts of strategy discovery, HPM is noteworthy for its level of specificity about mechanisms of strategy discovery and how those mechanisms produce new strategies. HPM includes 21 metacognitive reasoning heuristics, including a heuristic to identify redundant sequences of actions within a strategy, a heuristic to identify parts of a strategy that produce results irrelevant to the solution of a problem, and a heuristic to compare alternative procedures in terms of required cognitive effort.

As it solves problems, HPM maintains a complete record of its activity. The metacognitive agents constantly scrutinize this memory trace, searching for redundant or inefficient steps. When such steps are identified, the metacognitive agents specify how HPM should rewrite its strategy to eliminate the offending procedures.

From sum to min. HPM begins as an addition novice that knows only production rules that produce the sum strategy. From this beginning, the program's metacognitive heuristics make each of the three necessary insights in three discrete steps. First, the redundancy-detection heuristic notices that after it has represented the quantity corresponding to each addend, it invariably performs the same sequence of counts that it just completed when counting out the first addend. HPM transforms its addition strategy so that, rather than always beginning the count of the represented addends from 1, it counts from the value of the first addend. Now that the only reason to count out the first addend is to provide a starting place for the sum count, HPM is primed to achieve its next insight: The value of the first addend count is always the same as the number name of the first addend. HPM then decides that it no longer needs to count the first addend at all and rewrites its strategy so that the sum count begins at the value of the first addend.

The simulation is now on the verge of the final insight that will allow it to discover the min strategy. At this point in its development, it solves all arithmetic problems by counting-on from the value of the first addend. Eventually, HPM notices that a particular pair of addends always produces the same sum regardless of which addend is first despite the fact that the number of counts needed to compute a sum varies with the order of the addends. For example, the system notices that when solving 2 + 6, it makes six counts to get to 8, but when solving 6 + 2 it requires only two. The efficiency heuristics realize that it is always faster to begin counting-on from the larger addend, and they rewrite HPM's strategy to reflect this. The min strategy has been discovered, and is subsequently used on all problems.

Evaluating the fit of HPM to the microgenetic data. Similar to the children in Siegler and Jenkins (1989), HPM successfully makes the sum-to-min transition without ever using an illegal addition strategy. The program's production rules, metacognitive heuristics, and detailed memory for past problem-solving experience are carefully orchestrated so that each step in the discovery process is constrained to produce a stable, legitimate procedure for adding numbers.

However, to achieve these impressive constraints, HPM relies upon several assumptions that are inconsistent with the empirical findings. Probably the most serious inconsistency involved the hypothesized transition strategy of counting from the first addend. None of the children in Siegler and Jenkins (1989) used this strategy before using the min strategy, and only one child ever used it. Thus, it is not a good candidate as a transition strategy.

A second problem concerns the speed of learning. Similar to most metacognitive accounts, discoveries in HPM occur as classic ‘‘Eureka’’ moments: The system gains sudden insight into how it could transform its current approach to create a more efficient strategy. Once the new strategy has been created, its prior addition strategies are deleted and the new strategy is always used. Only some of the children in Siegler and Jenkins (1989) demonstrated any type of initial insight, and no subjects quickly generalized the new strategy.

A third problem involved the variability of discoveries. To successfully navigate the sum-to-min transition HPM must replace each of its arithmetic strategies in a particular order. Every incremental refinement creates the conditions necessary for the next refinement to occur. Each time the system is set in motion, the necessary sequence of refinements occurs in exactly the same order at exactly the same time. Such deterministic precision is inconsistent with the widespread variability that children exhibit as they discover the min strategy. Each of the children in the Siegler and Jenkins study exhibited a unique profile of strategy use leading to their discovery of the min strategy.

HPM’s strategy discovery machinery also makes substantial demands on cognitive resources. The metacognitive heuristics operate continuously; therefore, the system must always maintain sufficient attention and working memory to simultaneously support the heuristics as well as the execution of the strategy it is using to solve the current problem. At the same time, HPM is also faithfully recording a trace of all ongoing problem-solving activity and storing it in long-term memory. Although individuals might occasionally be able to coordinate these cognitive tasks, it seems unlikely that it is a common occurrence. It seems an especially daunting load for young children, whose processing speed is limited (Kail, 1991).

GIPS: An Associative Account of Strategy Discovery

Jones and Van Lehn’s (1991) *G*eneral *I*nductive *P*roblem *S*olving system (GIPS) illustrates a path to discovering the min strategy that relies primarily upon associative learning mechanisms. Like HPM, strategies in GIPS are implemented as the firing of a particular sequence of production rules. Also similar to HPM, at each step during problem solving, GIPS examines all of its rules to identify the one that best matches its current goals and the problem-solving context. However, GIPS does not calculate this match in an all-or-none fashion. Each rule in GIPS has a set of associative strengths linking it to particular elements of the problem-solving context. GIPS chooses rules by finding the one with the highest probability of being appropriate. These probabilities are defined as a function of the strength with which the rule’s conditions are associated with the current problem-solving context. After each successful use of a rule, GIPS strengthens the associations between the rule and the context. After each unsuccessful use, the associations are weakened.

From sum to min. When GIPS begins its run, its database is tuned so that it always chooses the sequence of rules that correspond to the sum strategy. As the system gains experience using this strategy, it extracts the same correlation that was noted first by HPM's metacognitive heuristic: The duplication between counting out the first addend and later counting the positions of the combined representation that corresponds to the first addend. Eventually, the associations that express this correlation become strong enough to cause GIPS to immediately fire the rules that constitute its sum count rather than first firing the sequence of rules it uses to count up the addends separately. At this point, GIPS has made the transition from the sum to the shortcut sum strategy. On a problem such as 2 + 3, GIPS puts up two ''fingers'' while counting ''1, 2'' and then puts up three more ''fingers'' while counting ''3, 4, 5.''

As it uses the shortcut sum strategy to solve problems, further regularities begin to emerge in the associative data base. First, GIPS learns that it is more likely to get a problem correct when the first addend is also the larger addend. The reason this is true is because the system is programmed to have a higher probability of miscounts when simultaneously keeping track of two counts, as is required when counting out the number corresponding to the second addend in the shortcut sum strategy. The larger the number of counts needed to represent the second addend, the higher the probability that a miscount during this phase will lead to a wrong answer. The correlation between addend size and error rates continues to strengthen until GIPS chooses to begin counting with the larger addend, regardless of whether it is first or second.

At the same time GIPS is learning about addend order, it also is learning the last associative regularity needed to discover the min strategy. Whenever the system counts out an addend, the last number it ''said'' in the counting sequence is always equal to the value of the addend itself. When this correlation has been sufficiently strengthened, GIPS begins to represent the larger addend by simply ''saying'' its value, and then counts on the number of times required to represent the smaller addend. At this point GIPS can be said to know the min strategy.

Evaluating the fit of GIPS to the microgenetic data. In at least two respects, GIPS is consistent with the microgenetic data from Siegler and Jenkins (1989). First, by using the shortcut sum strategy as a transitional step from the sum to the min strategy, GIPS mirrors the most common path identified by Siegler and Jenkins. Second, the gradual associative learning mechanisms at the core of GIPS require that the simulation solve many addition problems before it discovers the min strategy. This is consistent with the observation that most children discovered the min strategy after moderate amounts of practice in the domain.

However, in two other respects, GIPS falls short of a full account of strategy discovery. It does not exhibit strong constraints prior to discovery. In fact, GIPS cannot keep itself from making mistakes. As associative strengths fluctuate in GIPS, the simulation sometimes tries to execute a procedure that consistently generates wrong answers. To avoid going down such dead ends, GIPS is designed to stop and ask its human operator whether the procedure it is about to attempt is, in fact, a legitimate piece of an addition strategy. As Jones and VanLehn acknowledge, if GIPS were not told that execution was wrong, it would develop wrong strategies. Children do not have the luxury of having an expert monitor the legitimacy of the subprocedures within their strategies, yet they still manage to discover the min strategy without using illegitimate strategies of the type generated by GIPS.

GIPS also shows the too-rapid generalization exhibited by HPM. Both systems only use one way of solving addition problems at a given time. Once a new strategy is discovered, GIPS switches to using it exclusively. Jones and VanLehn note that GIPS' associative learning mechanisms are, in principle, capable of being modified to exhibit the same selection among multiple strategies as children show. However, even if GIPS were modified to maintain multiple strategies and to choose among them, the system would remain incapable of modeling the connection between increased awareness of the new strategy and increased generalization. There is no facility in GIPS through which metacognitive awareness could have any impact upon the associative mechanisms of strategy selection.

Summary

At the most general level, both HPM and GIPS successfully model discovery of the min strategy. Similar to children, the models begin by knowing how to use the sum strategy and discover how to use the min strategy. Also like children, neither HPM or GIPS requires direct instruction to discover the new approach. They both make the discovery based on what they learn about addition as they gain experience solving problems.

However, at the microgenetic level of analyses, neither the metacognitive mechanisms of HPM nor the associative mechanisms of GIPS can provide a complete account of the empirical data. Both approaches successfully model only one of the four main aspects of strategy discovery identified by Siegler and Jenkins (Table 1).

Like most metacognitive proposals, the mechanisms of HPM are well suited to generating legitimate discoveries. Its heuristics continually adapt strategies until they become optimally efficient. Because the metacognitive mechanisms have unrestricted access to a complete memory trace of prior problem-solving experience, they are able to prune strategies without ever lopping off parts of the procedure that are essential to the strategy's success.

TABLE 1
Comparing Empirical Evidence and Computational Accounts of the Discovery of the Min Strategy

Observed behavior of preschoolers	Metacognitive mechanisms of HPM	Associative mechanisms of GIPS
Discoveries occurred at varied times and on varied problems.	No	Yes
Discoveries occurred with varying degree of reflective insight.	No	No
Newly discovered strategies were never immediately generalized; however, children with greater reflective awareness generalized more rapidly.	No	No
Discoveries were constrained to avoid illegitimate strategies.	Yes	No

However, the constraints that make HPM a good model of the empirical data in one respect make it inadequate in other respects. Every time it is run, HPM's deterministic metacognitive mechanisms make the same sequence of discoveries at the same time. All of HPM's learning requires metacognitive insight. Once a new strategy is discovered, it is immediately generalized to all further problems.

The associative mechanisms of GIPS have a complimentary set of strengths and weaknesses. Like most associative models, discoveries in GIPS arise from a combination of experience and probability. Different runs of GIPS could discover the min strategy at different times and on different problems. However, the variation in *when* strategies are discovered does not carry over to variation in *how* strategies are discovered. GIPS has no metacognitive knowledge to enable it to achieve insight-driven discovery. The lack of metacognitive knowledge makes it impossible for GIPS to constrain its variation to legitimate approaches. Like most associative models, GIPS is, in the end, a trial-and-error learner. Finally, although the associative mechanisms of GIPS could be consistent with the gradual generalization of new strategies, a lack of metacognitive awareness leaves GIPS unable to model the finding that awareness of the new approach was related to its more rapid generalization.

Perhaps it should not be surprising that both empirical and computational research has failed to identify the *one* way that children discover the min strategy. Why would evolution have favored the phylogenetic development of humans so rigid as to have only a single path available for discovering problem-solving strategies? It makes more sense to think of strategy discovery as occurring through a varied set of mechanisms that can take advantage of whatever mix of associative experience and metacognitive understanding is available to an individual child at a particular point in development of a skill. In the remainder of this article, we sketch out what these mechanisms might look like.

STRATEGY USE AS COMPETITIVE NEGOTIATION

The empirical and computational evidence concerning discovery of the min strategy suggests the need for a new approach to conceptualizing the ways that metacognitive and associative knowledge interact in strategy discovery. Consistent with the approaches we reviewed, we propose that the domain specific knowledge involved in strategy use can be represented in different forms by a metacognitive system and an associative system. However, we propose further that metacognitive and associative versions of the same strategy can exist simultaneously and that strategy use emerges from a *competitive negotiation* between the two forms of the strategy.

What is competitive negotiation? Consider the example of a rider and her horse. Although horse and rider move through the environment as a single system, each has unique ways of representing and learning about the experience. If it is the first time they have been on a particular path, the rider will make most of the navigational decisions. She plans a route and uses the reins and spurs to encourage the horse to follow the plan. Although she can point the horse in a general direction and encourage the horse to move faster or slower, she does not specify the placement of each hoof. The horse scans the ground in front of it, continuously deciding how to coordinate the commands of the rider with the constraints of the terrain and its own physical limits.

When the horse becomes more familiar with the route, the rider no longer needs to provide continuous guidance because the horse has learned the choices that compose that path. Freed from the burden of navigating, the rider can now give the horse its head and relax and enjoy the scenery. However, if something catches the rider’s eye and she decides to deviate from the usual path, her high-level commands may initially be less potent than before. The horse has learned to follow one path and suddenly the rider is asking it to follow another. The animal may initially choose to ignore the novel commands, but it will find this an increasingly difficult task as the spurs dig deeper and the reins pull tighter.

The relation between a rider and a horse is a competitive negotiation. It is a negotiation because the pairs’ movement through the environment is defined by the overlap of the independent decision making processes of both individuals. It is competitive because the independent decision-making processes can come to different conclusions, and the result of the negotiation depends upon a test of wills. The rider and horse do not resolve differences by discussion; they do not seek intersubjectivity, and they do not try to create a cooperative solution. They act. It is the strength and insistence of their actions that determines which one decision will be the one that both follow.

The relation between metacognitive and associative knowledge can also be described as a competitive negotiation. The metacognitive and associative systems maintain separate representations and decision making processes. Metacognitive knowledge is explicit and potentially verbalizable and may be expressed as production rules, plans, or heuristics. Associative knowledge is implicit and non-verbalizable and might be best expressed as statistical associations among units of action and problem solving contexts. Although they exist in the same mind, and although they are simultaneously working on the same problem, the two systems are representationally encapsulated. That is, neither system can directly inspect or modify the contents of the other. Metacognitive and associative knowledge only interact through the output of independent encoding and decision-making processes.

At each problem-solving step, the systems independently encode the problem, match their representation of the problem to their respective knowledge bases, and select appropriate problem solving actions. Behavior is directed by the first system to produce a strongly supported decision about what to do next.[2]

The outcome of this competition is in large part determined by how much relevant, domain-specific experience the associative system includes. When the problem solver is working in familiar contexts, the fast, efficient associative knowledge base often produces satisficing decisions before the cumbersome metacognitive system reasons its way to a solution. However, in novel contexts where the associative system has not been tuned by experience, it can fail to produce a confident decision about what to do next. Such impasses open a window for the slower, though more broadly applicable, metacognitive system to reason its way to the next problem-solving action.

The outcome of the competition can also be determined by the goals adopted by the metacognitive system. In familiar contexts, where the associative system easily wins competition to direct processing, the metacognitive system is relieved of the burden of micromanaging strategy use. In these cases, it is free to give the associative system its head and focus on something else. The metacognitive system may focus on monitoring the progress of the associative system, checking the partial products of problem solving to make sure that no unexpected obstacles arise that would require its intervention. Such situations may also provide opportunities for the metacognitive system to notice and encode interesting aspects or concomitants of strategies that are not necessarily related to the immediate goal of solving the problem.[3]

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[2] The astute reader will wonder why the horse-and-rider analogy does not lead immediately to a classic homonculous problem. Although the metacognitive system (the rider) is indeed a partially independent system from the associative system (the horse), it is a self-contained system, and so needn't itself have a meta-meta-cognitive system, etc. Therefore, although we are describing an architecture with, so to speak, two minds, is has *precisely* two minds and not an infinite number of them.

[3] The metacognitive system may also decide to focus on something other than the problem at hand or the strategy being used. One example would be when a driver's associative system handles the actions necessary to operate the vehicle while his metacognitive system carries on a conversation with the passenger. Another would be a reader whose associative system

If monitoring or noticing leads the metacognitive system to perceive a need to intervene, it can increase its ‘‘bid’’ to control the problem-solving process by altering problem solving goals. Possible triggers for such intervention could include, among other things, the metacognitive system noticing something interesting about prior solutions, becoming tired of using the same approach, perceiving a time-saving shortcut, or encountering explicit instructions (e.g., from a teacher) about how to solve a problem. In such cases, the metacognitive system may adopt the goal, not just of solving the problem, but of solving it in a particular way.

By adopting this variation on the typical problem solving goal, the metacognitive system can influence the associative system to produce variation in strategy use that would never arise as a result of purely associative competition. It is important to note, however, that the input of associative mechanisms is never excluded from the decision-making process. Like a rider on a horse, the metacognitive system can suggest directions, but the associative system (the horse) remains the engine of problem solving. Increased metacognitive control shapes the associative landscape but can never replace it.

This view of strategy use as a process of competitive negotiation has the potential to account for changes in strategy use over time that are consistent with the four crucial characteristics of children’s discovery of the min strategy.

Discoveries Occur at Varied Times and on Varied Problems

When children are novices in a domain, and no strategy has become well established, the metacognitive system must explicitly decide to carry out each step in a strategy, sending commands to the associative system that executes these steps. With experience, the associative mechanisms begin to learn an automatized analog of the metacognitive version of the skill. As the efficient selection processes of the associative system gain primary control over behavior, the role of the metacognitive system increasingly is to observe and learn.[4]

If it notices something interesting, the metacognitive system can choose to exert supervisory control, resulting in an unusual competition between

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carries on processes of decoding and scanning a page of text while the metacognitive system day-dreams. Note, however, that in both of these examples, the disconnect is not complete. Treacherous driving conditions trigger increased metacognitive involvement in driving and a lull in the conversation. Eventually, the metacognitive system notes that the reader does not remember anything from the preceding paragraphs and directs the hand to turn back a page; the eyes to refocus on the beginning of the section, and the scanning and decoding to begin again.

[4] It is worth emphasizing that in the competitive negotiation theory, as contrasted with translation approaches, the skill is learned by the associative system through its experience on the task, not through any exchange of knowledge with the metacognitive system. As with the horse and rider, the rider cannot just tell the horse the route, she must ride the horse through it a number of times.

metacognitive and associative mechanisms. Choices that had most recently been made by associative mechanisms alone must now be made through a combination of bids from both systems. Due to this additional competition, and due to the slow and resource-intensive nature of the metacognitive system's control processes, problem solvers will show unusually long response times as they approach the discovery of a new strategy. The problem solver may be in a state of flux over several trials until the struggle between metacognitive and associative mechanisms finally works itself out.

Because these unusual competitions were triggered by noticing rather than impasses, they would be most likely to occur on problems with which children are familiar. Familiarity increases the likelihood that children have developed an associative version of an appropriate strategy, thus increasing the likelihood that they have sufficient metacognitive resources free to observe and notice. Familiarity also increases the likelihood that children have metacognitive experience that can be used to compare solutions to the same problem over time, thus increasing the likelihood that they will notice something interesting.

Discovery by metacognitive noticing will not occur at the same moments in different individuals. When discoveries are driven by impasses, they occur predictably: If children do not know an applicable strategy, then they must invent a new one. However there are not such clear criteria for when an observation has become sufficiently interesting to merit the creation of a new strategy. Observations are probably developed over several trials where a strategy was used successfully. The speed and the detail with which observations are developed depends on a range of factors. What particular problem solving tasks has an individual encountered? Has an individual's metacognitive system mostly focused on prior strategy use or has it been daydreaming while the associative system orchestrates problem solving? Does an individual remember a strategy that she observed another child using, or does she perhaps remember something a teacher or a parent told her?

Any and all of these factors, among others, are potential contributors to the metacognitive system deciding to intervene in problem solving. Because these factors vary widely between individuals—even individuals who are participating in the same microgenetic study—the moment at which the metacognitive system chooses to exert supervisory control will be correspondingly varied.

Discoveries Occur with Varying Degrees of Reflective Insight

A system based on competitive negotiation makes discoveries with a range of metacognitive insight. The knowledge of the metacognitive system is potentially verbalizable. When Siegler and Jenkins asked children for immediate retrospective verbal protocols, they were, in terms of this approach, asking for a dump of the metacognitive system's working memory. The quality of retrospective reports depends on the extent to which metacognitive mechanisms had been recently activated and used in the discovery process.

The metacognitive system can make bids of varying strengths to exert its supervisory control. Sometimes, a weak bid from the metacognitive system will be enough to nudge the associative system to select a new path. In these cases, there would be little metacognitive knowledge available to be reported because little metacognitive knowledge was involved in the discovery. If the metacognitive system takes a larger role in directing discovery of the new strategy, there will be a greater wealth of recently activated knowledge available to be included in the retrospective protocol. In the extreme case where the metacognitive system's bid is so strong that it effectively micromanages each subgoal in a problem solving strategy, the problem solver would be able to report a trace that outlines each procedure used to solve the problem.

Regardless of the level of metacognitive involvement in any particular discovery, new strategies would not be immediately generalized to all possible problems. In domains where children have at least a moderate amount of experience, the faster associative mechanisms are typically able to win competitions over the slower metacognitive mechanisms. Thus, on the trial following a discovery, a newly emerged strategy would be guaranteed of being chosen only if it could win competitions with established competitors on the associative level. This is a difficult challenge for a strategy that has a track record of just one use. Over time, if the newly discovered strategy provides a reliable advantage over older strategies, it comes to dominate the competition. But initial uses of a newly discovered strategy would be expected to be occasional at best.

Greater Reflective Insight Is Associated with Faster Generalization of the New Strategy

Although associative mechanisms often play the greater role in determining which strategies are selected, it is not the case that metacognitive mechanisms have no influence. When children's strategy discoveries involve greater input from metacognitive mechanisms, children create a richer metacognitive knowledge base about the new strategy. This knowledge base may include rules about when the new strategy is most useful, or memories about why a strategy was efficient or fun to execute. Acting on this knowledge, children may adopt the goal of solving a problem with a particular strategy, rather than solving a problem with whatever strategy is suggested by the associative system. This will lead to metacognitive insight about use of a new strategy being associated with more rapid generalization of it to new problems. For example, metacognitive mechanisms might notice that a problem has addends that differ widely in size (e.g., $2 + 9$), might remember that it once used a strategy that minimized counting (the min strategy, for instance), and might set the goal of solving this particular problem with that particular strategy.

Thus, although most strategy selection is determined at the associative level, metacognitive mechanisms may sometimes take control. The richer the metacognitive knowledge base about a strategy, the more likely that mostly metacognitive strategy selection will take place. With each use of a helpful new strategy, the associative basis for its use will strengthen, and the strategy will win more competitions in the future. Thus, even occasional metacognitive uses of a strategy can provide the boost that a new strategy needs to become a serious contender to win competitions through its associative strength.

Discoveries Are Constrained to Avoid Illegitimate Strategies

As associative mechanisms assume more responsibility for directing strategy use, and the metacognitive system is free to observe, the initial metacognitive version of the strategy begins to decay. In well-practiced domains, the metacognitive system will not often mount a serious challenge to associative selection; thus, the metacognitive rules that specify how to execute a strategy will eventually stop being activated in the course of normal problem solving. The metacognitive system, like the associative system, is a part of normal human memory. Elements in memory that receive little activation are harder to recall than elements that are regularly activated (Anderson, 1990). Eventually, rarely activated memories may be forgotten.

It is this process of forgetting metacognitive knowledge that leads to the creation of goal sketches. Recall that goal sketches enable children to recognize legitimate novel strategies even before they learn how to use the strategy, and even if they cannot explain the basis for that recognition. When they are complete novices at addition, children do not possess goal sketches for arithmetic strategies. If forced to add, they would not exhibit constrained discovery. They would have no recourse but to fall back on using unconstrained trial-and-error.[5]

The process that eventually produces a goal sketch begins when children learn their first addition strategy. Consider a child who is explicitly taught to execute the sum strategy. If the child learns the procedures correctly, the entire sum strategy is represented as a complete series of subgoals in the metacognitive system. When the child uses the sum strategy, this sequence of subgoals micromanages the associative version of the strategy.

As the child accumulates experience, associative mechanisms gradually assume responsibility for selecting and executing the procedures of the sum strategy. As it loses competitions to control behavior, the metacognitive system has little to do but observe the output of the associative system, possibly looking for interesting regularities or checking the partial products of the

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[5] Although complete novices do not have addition goal sketches, it is not the case that they know nothing relevant about legitimate addition strategies. By the time they learn to add, most children are already relatively skillful at counting—a vital skill in simple addition. Children who utilize counting in addition strategies will be able to call upon established counting associations: between 1 and 2, 2 and 3, and so on. When inventing addition strategies, these children would not be expected to violate counting principles. However they would be expected to make errors in how they orchestrate the counting procedures.

associative system's strategy choices (e.g., whether the first addend was represented.) Neither of these activities require the metacognitive system to activate its entire representation of the strategy, but they will often require that the metacognitive system activate the primary subgoals of the strategy. The metacognitive mechanisms would have difficulty monitoring the execution of a strategy or noticing interesting properties of it unless they were also able to carve the strategy into meaningful units. Thus, the metacognitive system would continue to activate the main subgoals of its representation of the strategy, even while the rest of the representation was atrophying from disuse.

This process of differential activation would accelerate as children learn a greater number of addition strategies. As previously noted, although legitimate addition strategies vary in their constituent procedures, they all share three essential subgoals: To quantitatively represent the first addend, to quantitatively represent the second addend, and to quantitatively represent the sum. The metacognitive representations of these shared elements could receive activation when any legitimate strategy is used. At the same time, the metacognitive representations of elements not shared by all strategies would receive less activation as children use a greater number of arithmetic strategies.

The outcome of this process is inevitable. Eventually, differential activation causes the essential skeletal structure of a strategy to emerge from what started as a complete metacognitive specification of a strategy. At this point the problem solver has developed a goal sketch.

This process of sculpting constraints through forgetting is the fundamental reason why goal sketches are useful in guiding discovery. When the metacognitive representation of specific procedures decay, but the representation of their goal structures remain, children become able to constrain discovery of new procedures and evaluate novel procedures that they encounter by comparing them to the goals of the task. If the metacognitive system retained the detailed description of the sum strategy, it would provide too many constraints to allow any new discovery. In short, the power of goal sketches is that decay of metacognitive representation of component procedures provides freedom to generate new procedures that are constrained to meet basic task goals but that can differ from the original procedure in how they do so.

HARDENING THE CORE

Klahr (1992) distinguished two levels of theorizing about cognitive mechanisms. In soft-core theories, mechanisms and hypotheses are expressed verbally, and perhaps graphically. In hard-core theories, the verbal and graphical representations are translated into the code of a running computer simulation. Both soft and hard core theorizing have their place in developmental psychology. However, we agree with Klahr that producing running computer simulations is a crucial step in theorizing because they provide a rigorous test of the internal consistency of the model and of its sufficiency to produce the intended behavior. Unexpected behavior of such simulations may also spark new predictions and experiments to test those predictions (Siegler & Shipley, 1995).

In this article we have a outlined a soft-core model for strategy discovery though competitive negotiation. In doing so, we have begun with empirical constraints, explored alternative models, and proposed a set of alternative mechanisms that we believe are sufficient to match the empirical record. However our soft-core proposals are only the first step in fully developing a model of competitive negotiation. We are currently completing the second step by developing a computer simulation model of min strategy discovery through competitive negotiation. In developing a running model, we need to answer the questions raised by the verbal description of the model we have presented here. These include: What is the exact arithmetic knowledge represented in the metacognitive and associative systems at different points in development? What functions and parameters are necessary to implement competitive negotiation? How much experience is required to learn an associative version of the sum strategy? How exactly does a metacognitive plan fade into a goal sketch?

Artificial Intelligence researchers refer to architectures that combine symbolic and associative components as ‘‘hybrid models’’ (Gutknecht, 1992; Medsker & Bailey, 1992). Since the first, we have conceptualized, and more importantly implemented, strategy use in hybrid terms. Both the Siegler and Shrager (1984) and the Siegler and Shipley (1995) models are hybrid models in the sense that symbolic problem-solving trains an associative memory for arithmetic facts; no direct translation from one representation to the other takes place even though these models learn to give answers more efficiently. The resulting associations compete with the strategies to provide answers. Unfortunately, neither of these models is capable of strategy discovery in the way that we have described it because in these models the strategies themselves are not decomposed and distributed through the associative memory. However, they provide a starting point for our current simulation effort in which we combine metacognitive discovery processes, similar to those found in HPM, with an associative component that can represent the constituent procedures of strategies as well as just arithmetic facts (as was done in Siegler & Shrager, 1984) or facts and the statistical selection criteria for undecomposed strategies (as was done in Siegler and Shipley, 1995).[6]

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[6] ACT* (Anderson, 1983) represents problem-solving strategies in decomposed form and in both declarative (approximately meta-cognitive) and procedural (approximately associative) forms. However, the process of ‘‘proceduralization’’ (approximately compliation) in ACT* is one of *translation* from declarative to procedural which is supposed to happen internally as a result of practice. Furthermore, neither ACT*, nor any other cognitive model we are aware of, has separate execution mechanisms that could enable competetive negotiation.

Aside from the usual challenges of specifying the theoretical nuts and bolts, we expect that simulating strategy use as competitive negotiation will help us confront a more general issue. The recent proliferation of microgenetic studies has begun to change the field's understanding of what it means to make a discovery. Based on recent evidence, it seems clear that the characteristics of discovery revealed in the Sielger and Jenkins study are not limited to simple arithmetic. Across a wide range of domains and ages, strategy discoveries occur on varied problems and at varied times, occur with varying degrees of reflective insight, are generalized slowly, and are constrained in ways consistent with goal sketches (Kuhn, 1995).

These data converge to create a new view of what it means to discover a strategy. Discovery has often been conceptualized as an elevated, special form of reasoning, used primarily or exclusively when problem solvers were at impasses. The resolution of those impasses was often described as involving a sudden flash of insight, or as the product of careful metacognitive reasoning. This perspective, however, does not fit the data on children's discoveries, either on familiar tasks, such as making change and solving arithmetic problems (Lawler, 1985; Siegler & Jenkins, 1989), or on novel tasks, such as scientific experimentation and map-making (Karmiloff-Smith, 1992; Kuhn, Garcia-Mila, Zohar, & Anderson, 1995; Schauble, 1990).

Nor do existing computer simulations of discovery appear sufficient to account for the phenomena. The problem goes beyond the models we have reviewed here to encompass all models that would rely exclusively on metacognitive insight or exclusively on associative mechanisms. Simply put, models based only on metacognitive mechanisms, or only on associative mechanisms seem insufficient to explain the complex mixture of insightful and uninsightful behavior that the microgenetic studies have revealed. To cite one example, they seem insufficient to explain how children can often, but not always, exhibit substantial metacognitive understanding on their first use of a new strategy, go on to generalize the strategy slowly regardless of the degree of understanding exhibited, but extend it somewhat more rapidly if they showed insight than if they did not. In general, to account for people's discoveries, the next generation of models will need to account for more variability within individuals, more variability across individuals, and more variability across contexts. We believe that conceptualizing strategy use as a competitive negotiation between metacognitive and associative knowledge is a step toward this goal.

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